DOCUMENT RESUME

ED 057 481 EA 003 892

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TITLE Economies of Scale for a State System of Public School Districts.
PUB DATE [71]
NOTE 28p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Centralization; *Cost Effectiveness; Decision Making; Economic Factors; Economic Research; *Educational Economics; Educational Planning; Financial Policy; *Models; *School Districts; State Departments of Education; *Statewide Planning

ABSTRACT

This paper presents a decision model based on microeconomic theory. The model utilizes a stepwise polynomial regression to derive a cost function and then constructs an average cost curve to identify an optimal school district size. An application of the model to the State of Oregon illustrates how economies of scale information for a State system can be used by policymakers to evaluate proposed district consolidations in light of opportunity costs. Data inputs for the model are available in the published records of State departments of education, which facilitates application of the model in other States. (Author)

ED057481

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ECONOMIES OF SCALE FOR A STATE SYSTEM OF PUBLIC SCHOOL DISTRICTS

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A prevailing trend in educational planning over the past thirty years has been a movement toward the formation of larger and larger school districts. Simon and Grant (1968) have shown that the number of public school districts in the United States decreased from 127,531 in 1930 to 54,859 in 1955 and continued to decrease to 26,983 in 1966. An examination of more recent educational planning activities such as (1) the preparation of state-local program planning systems by State Departments of Education, (2) the development of regional instructional resource centers, and (3) the emerging number of intermediate units in the various states, lends additional evidence that this trend toward larger administrative units and school districts still prevails and is a current concern of educational planning specialists at all institutional levels.

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In light of this trend economists and educational planners must continue to ask the question, "Is there still an economic justification for larger school districts and if so is there some point at which the economic advantage disappears?" The model developed here is designed to provide better descriptive information to support policy makers who must not only answer this question but must also provide state boards of education and local school district administrators with alternative strategies or plans to implement prescribed policies.

The model is explicitly designed to use statistics that are now collected by state departments of education. Hence it should provide a generalizable and inexpensive procedure that can easily be adapted within any state. In this study the properties of the model are illustrated using data on record in the Oregon Department of Education.

ECONOMIC MODELS

A number of models dealing with the economies of scale in the educational sector have appeared in the literature. Common to most of these studies is some form of regression analysis. This approach is used by Hirsh (1960) who developed least squares expenditure functions to analyze economies of scale for 27 school districts in the St. Louis City-County area. In his model, total current expenditures are regressed on average daily attendance and various demographic variables including an "index of scope and quality" (pp. 33-35). Total current expenditures plus debt service are also regressed on the

same set of independent variables. His findings indicate that no significant economics of scale could be detected for this geographic area, and hence no economic justification for the consolidation of these school districts into larger units.

Riew (1966), in analyzing the relationship between school size and costs, focuses on Wisconsin public senior high schools rather than all types of school districts. His sample is further restricted since school districts having more than one senior high school are excluded. He regresses operating expenditures per pupil on average daily attendance and a selected number of other input measures. Operating expenditures in his model excludes outlays for transportation, school lunch, pupil recreation, health programs and other "minor" items (p. 282). Since he assumes a U shaped average cost curve, at some point diseconomies of scale do set in.

Osborn (1970) has recently examined the relationship between costs and school district size for 433 districts located in the state of Missouri. Unlike the Riew study, Osborn's sample includes districts that have high schools as well as elementary units. His criterion variable is current expenditures per pupil. Independent variables include a measure of school classification, tax levy, district wealth, geographic location, and teacher salary. He also assumes a parabolic relationship between per pupil expenditures and enrollements (p. 114).

Although Osborn and Riew both determine a point at which diseconomies of scale set in, there exists a large discrepancy in their

rates of per pupil expenditures. Osborn suggests one possible explanation for this is that Riew excluded transportation costs from his expenditure variable.

Another approach used by Cohn (1968) is to first construct a production function

$$Q = f(X_1, X_2 \ldots . X_n)$$

plus a budget constraint

$$C = \sum_{i=1}^{n} P_i X_i$$

where X_1 represents the quantity of the factor i in the production relationship and P_i is the price of factor i. The production function can then be solved for one variable, say X_n in terms of $X_1, X_2, \ldots, X_{n-1}$ and Q. This equation is

$$X_n = h(Q, X_1, X_2, \ldots, X_{n-1})$$

Substituting this function into the budget expression yields the following cost function

$$C = g(Q, X_1, X_2, \ldots, X_{n-1}).$$

Average cost is then regressed on $Q_1, X_1, X_2, \ldots, X_{n-1}$ and the partial derivative $\partial AC/\partial Q$ is set to zero to find the optimum scale.

One limitation with this approach is that the model builder must completely specify the production function, that is, all of the relevent inputs need to be identified and measured accurately in order to explain the total variance within the school system.

It is our contention that this approach, while theoretically correct, for all practical purposes complicates the problem. If the assumption that school districts operate in an equally efficient manner (what else can one assume when doing an economies of scale study?) is reasonable, then we believe that the cost function can be simply expressed as a function of output alone. Kosobud (1963) argues that in determining economies of scale for the public sector "the problem for school officials is formulated to be that of minimizing variable costs for a given enrollment in an educational program of a given content"(p. 260).

A brief description of the different variables, school district samples, regression techniques and a priori assumptions utilized in the models above clearly indicates that no standard model has been developed for the problems of economies of scale within the educational sector.[1] Based on the analysis of previous research efforts, we offer a model that can easily be applied to statistics now available in State Departments of Education, and which at the same time provides valuable new information about the existing cost-size relationships and opportunity costs that exist in a state system of school districts.

DEVELOPMENT OF THE MODEL

Consider a school district with the following production functions:

$$Q = f(X_1, X_2, \ldots, X_n) \qquad (1)$$

where Q is the number of students who are in residence and utilizing the services of a school district. Let X_i be the input variables or resources such as teachers, instructional materials, curriculum co-ordinators, physical facilities, etc. The production function is

therefore a mathematical expression indicating the way in which various inputs are combined to produce the desired output (see Bowles, 1970). The district's budget can also be expressed in terms of resources such that

$$C = P_1X_1 + P_2X_2 + \ldots + P_nX_n \qquad (2)$$

where P_i is the price of the input X_i.

The district then attempts to minimize its required budget subject to the constraint that it must provide appropriate educational opportunities and supporting services for Q° students in a various number of instructional program activities. To reveal the consequences of cost minimizing behavior, a function may be formed from the budget constraint and the production function by introducing a Langrangian multiplier. This function would be

$$\text{Minimize } C = \sum_{i=1}^{n} P_iX_i - \lambda[f(X_1, X_2, \ldots, X_n) - Q^\circ] \qquad (3)$$

The Lagrangian expression could then be solved to find the optimal values $X_1^\circ, X_2^\circ, \ldots, X_n^\circ$ that satisfy the production function and minimize the required budget C (see Baumol; 1961, p. 63).

The Two Variable Case

For illustrative purposes assume that there are only two inputs X_1 and X_2, then our production function becomes $Q = f(X_1, X_2)$ and the budget expression is $C = P_1X_1 + P_2X_2$. The solution to the Lagrangian expression below,

$$\text{Minimize } C = P_1X_1 + P_2X_2 - \lambda[f(X_1, X_2) - Q^\circ] \qquad (4)$$

may be illustrated in diagramatic form as in Figure 1.

Insert Figure 1

The price parameters P_1 and P_2 determine the slope of the budget expression, but since C is a variable there exists an infinite number of possible lines such as C_1 and C_2 with the same slope $(-P_1/P_2)$ but with a variable intercept having a value of (C/P_2). The minimum C_o is determined at the tangency of the production function isoquant Q° at point E with coordinates (X_1°, X_2°).

To extend the two variable model, assume now that there exists an infinity of school districts, each with similar production functions and each facing the same input prices. The result of this cost minimizing behavior for a finite set of these districts is illustrated in Figure 2. The locus of tangencies trace out what is called an efficiency locus $g(X_1, X_2) = 0$.

Insert Figure 2

For every point $(\hat{X}_1, \hat{X}_2)$ on the efficiency locus there corresponds a unique set $(\hat{C}, \hat{Q})$ that is, cost can be expressed as a function of quantity (see Henderson and Quandt; 1958, p. 55). Hence in this system there are three equations

$$Q = f(X_1, X_2) \tag{5}$$

$$C = P_1X_1 + P_2X_2 \tag{6}$$

$$0 = g(X_1, X_2) \tag{7}$$

with four variables C, Q, X_1 and X_2.

This system can be solved as follows. Based on (7), X_1 can be expressed in (8) as a function of X_2. Hence,

$$X_1 = h(X_2) \tag{8}$$

Now (8) can be substituted into (5) and (6) to yield

$$Q = f(h(X_1), X_2) = H(X_2) \tag{9}$$

and

$$C = P_1h(X_2) + P_2X_2 = F(X_2) \tag{10}$$

The production function in (9) can now be solved for X_2 such that

$$X_2 = H^{-1}(Q) \tag{11}$$

Finally by substituting (11) into (10) a cost function can be determined that expresses school district expenditures as a function of output defined to be the number of students who will utilize the instructional resources and other services provided by the schools. This function is

$$C = F(H^{-1}(Q)\ ) = R(Q) \tag{12}$$

A graphic illustration for (12) appears in Figure 3. The cost function, C = R(Q), has a positive slope as depicted in Figure 3. This should follow from a close examination of Figure 2. Notice that for every ascending pair of values (X_2^0, X_1^0), (X_2^1, X_1^1), ..., (X_2^n, X_1^n) there corresponds a unique set of cost-quantity values (C_0, Q_0),

$(C_1, Q_1), \ldots, (C_n, Q_n)$ with $C_0 < C_1 < \ldots < C_n$ and $Q_0 < Q_1 < \ldots < Q_n$.

Insert Figure 3

The average cost curve, AC, can now be determined by dividing each side of (12) by Q such that

$$AC = C/Q = R(Q)/Q \tag{13}$$

This curve is used to determine if an optimal school district size does exist. Figure 4 illustrates a case where the scale of operations can be expanded upward to Q* with cost per student unit declining, but after Q* average costs increase. Hence, Q* would be the optimal school district size. Figure 5 illustrates a situation where no feasible optimal size could be determined since this type of curve seems to imply the larger the school district the better.

Insert Figures 4 & 5

The Three Variable Case

In the two variable example above, the cost function and the average cost curve are derived from a system of three aquations with four variables. For the three variable case from (1) and (2), a system of four equations in five variables can be formed so that the average cost curve and the cost function can also be determined.

For three input variables the Lagrangian expression would be

$$\text{Minimize } C = \sum_{i-1}^{3} P_i X_i - \lambda[f(X_1, X_2, X_3) - Q^\circ] \tag{14}$$

The first derivatives with respect to each variable can now be constructed such that

$$\partial C/\partial X_1 = P_1 - \lambda f_1(X_1, X_2, X_3) \tag{15}$$

$$\partial C/\partial X_2 = P_2 - \lambda f_2(X_1, X_2, X_3) \tag{16}$$

$$\partial C/\partial X_3 = P_3 - \lambda f_3(X_1, X_2, X_3) \tag{17}$$

$$\partial C/\partial \lambda = f(X_1, X_2, X_3) - Q^\circ \tag{18}$$

Setting each derivative equal to zero leaves

$$\lambda = P_1/f_1(X_1, X_2, X_3) \tag{19}$$

$$\lambda = P_2/f_2(X_1, X_2, X_3) \tag{20}$$

$$\lambda = P_3/f_3(X_1, X_2, X_3) \tag{21}$$

This indicates that in the optimal solution all three coefficients in (19) through (21) are proportional. Hence we can construct two equations

$$0 = g_1(X_1, X_2, X_3) \tag{22}$$

$$0 = g_2(X_1, X_2, X_3) \tag{23}$$

Equations (22) and (23) thus express the general efficiency locus for

3 factors (under constant factor prices). It represents a one-dimensional curve running through the 3 dimensional factor space. (See Frisch, 1965, p. 161).

For the three variable case the following system of equations could be used to determine C and AC. These are

$$\begin{aligned} Q &= f(X_1, X_2, X_3) \\ C &= P_1X_1 + P_2X_2 + P_3X_3 \\ 0 &= g_1(X_1, X_2, X_3) \\ 0 &= g_2(X_1, X_2, X_3) \end{aligned} \tag{24}$$

Hence the three variable example has one more variable and equation than does the two variable case.

The General Case

For the solution to the general Lagrangian expression in (3), Frisch has shown that there exist n - 1 equations that express the efficiency locus for n factors. As in the three variable case above, the locus represents a one-dimenisonal curve running through n dimensional factor space. These equations would be

$$\begin{aligned} 0 &= g_1(X_1, X_2, \ldots, X_n) \\ 0 &= g_2(X_1, X_2, \ldots, X_n) \\ &\vdots \\ 0 &= g_{n-1}(X_1, X_2, \ldots, X_n) \end{aligned} \tag{25}$$

These n - 1 equations plus (1) and (2) now form the system of equations that can be used to determine the cost function and the average cost curve for a set of school districts. In the general case, it should be noted that the system has n + 1 equations with n + 2 unknowns. The solution to this system yields a cost function and an average cost curve that are both functions of Q, the number of students who are in residence and utilizing the services of a school district.

It should be obvious that to implement such a model for a large number of school districts within a state for a given year would involve an infinity of calculations. In general, this would still be true even if nonlinear programming methods are used to solve the system of n + 1 equations. However, if it is assumed that each school district's expenditure record reflects an equivalent pattern of efficiency, one can move toward approximations for the cost-size relationship with available data on C, current operating expenditures, and Q, the average daily membership (ADM) for each district in the model.

Based on the assumption above and the previous analysis, it is suggested here that the cost function derived in (12) can be easily and directly approximated to satisfy current planning needs by means of a step-wise polynominal regression equation. This method does not require the a priori assumption that there exists a parabolic arc which is negative in the linear and positive in the quadratic term. Hence,

$$C = b_0 + b_1Q + b_2Q^2 + \ldots + b_nQ^n \qquad (26)$$

Sabulao and Hickrod (1971), in their analysis of economies of scale for school districts in Illinois, use a similar regression approach to estimate a cost-size function. Unlike Riew and Osborn, they do statistically test departures from linearity rather than merely assume that they exist (pp. 8-9). They use a number of different transformations such as log functions to derive the best least squares fit.

The average cost curve can be calculated as in (13) by dividing through (26) by Q so that

$$AC = b_0Q^{-1} + b_1 + b_2Q + \ldots + b_nQ^{n-1} \qquad (27)$$

The optimal school district size corresponds to the minimum of the average cost function as shown in Figure 4. This can be determined graphically or more accurately using differential calculus. In this latter approach one finds the root of

$$\partial AC/\partial Q = 0 \qquad (28)$$

Where(28) represents the first derivative of AC. This root is the optimal school district size, that is, the point at which the average cost per student no longer declines as the ADM increases.

APPLICATION OF THE MODEL

In this section the model is applied to data published by the Oregon State Department of Education for the 1968-69 school year. The school district sample includes all unified school districts in the state that have an average daily membership (ADM) of greater than 100 students.

Unified districts are defined as districts that contain grades 1 through 12. This type of district is used for our sample because recent consolidations within the state have resulted in an increase in unified rather than other types of districts. Proposed district mergers in general also favor the unified district concept. Moreover, since the flow of students from elementary to secondary levels is an intrinsic operation in unified districts, similar ratios should exist for the student populations in each level over time.

The cost function for these 142 Oregon school districts is estimated by means of the step-wise polynominal regression described in (26). For computational efficiency (26) is modified so that

$$Z = B_o + B_1A_1 + B_2A_2 \ldots + B_nA_n \qquad (29)$$

where $Z = (10^{-5})\ (C)$ and each $A_i = (10^{-2i})\ (Q^i)$. This transformation allows the computer program to maintain accuracy for the higher order polynominal terms.

Table 1 shows the results for (29). The analysis indicates that no improvement in the sums of squares can be made once the third degree polynominal is fitted. The lower portion of Table 1 shows the coefficients for (26) and (27), the cost function and the average cost curve.

Insert Tables 1 and 2

Newton's approximation method is used to solve (28). This

solution, corresponding to the minimum of the average cost curve, is in the neighborhood of 51,000 ADM. A strict interpretation of the point of optimum scale is somewhat difficult, since the average cost curve is very flat in the range from 40,000 to 60,000. This can be observed in Figure 6 as well as in Table 2. For policy planning purposes based on this criterion of efficiency, any district within this range could be viewed as optimal.

These results seem to lend some support for the model system of federated school government suggested by Nystrand and Cunningham (1970). In this type of administrative unit, called a metropolitan district, there would be a series of operating local districts. They note " in ideal terms, each of these (local)districts would be socially, economically, and racially heterogeneous and enroll from 25,000 to 40,000 students" (p. 105). These results also lend some support for similar school district size recommendations provided in other recent studies. These are summarized in Hickey (1970, p. 30) and are grouped by criterion variables such as community control, general quality, not current expenditures, administrative decentralization, special services, and tax effort required.

Policy Planning

The following example illustrates how the results obtained above could be used in policy planning. Within a large metropolitan area in western Oregon having a total population of 120,000 there are

three adjacent unified school districts with current average daily memberships as indicated in column one of Table 3. All three districts are also members of a regional administrative unit, in this case called an intermediate educational district. If these districts wished to consider a consolidation as a means to expand the current level of educational services provided to their students as well as to reduce the overhead cost for each individual district, the cost information provided in Table 3 could be used as one measure to evaluate the economic consequences of this proposal.

Insert Table 3

Column two contains the appropriate entry for each district and the proposed consolidation membership based on the average cost curve found in Table 1. These estimates are used rather than actual per pupil costs, since, in any particular year, a district's per pupil expenditure may reflect some nonrecurring cost considerations. Such expenditures are not uncommon due to the fact that school district accounting entries for state reports are usually recorded on a cash rather than an accural basis (for example see the U.S. Office of Education accounting procedures for school districts described in Mort, Reusser and Polley, 1960; Chapter 20).

This analysis indicates that the consolidation proposal could result in a substantial savings as illustrated in column four. Based on these favorable results, policy planners might now examine other

socio-economic information for their region to determine if similar gains would follow. (For example see Advisory Commission on Intergovernmental Relations, 1965; Hickey, 1970; Nystrand and Cunningham, 1970). Economies of scale models (and this one's no exception) are not designed to replace judgment on the part of policy makers. Their explicit purpose is to provide policy makers with information about the future consequences of particular organizational alternatives in light of existing cost-size relationships and opportunity costs.

SUMMARY

In this article we have presented a decision model to be used in connection with data that are currently available in state departments of education to determine economies of scale for a state system of public school districts. The model, based on micro-economic theory, utilizes a step-wise polynominal regression to derive a cost function and then to construct an average cost curve that can be used to indentify an optimal school district size.

An application of the model to the state of Oregon illustrates how economies of scale information for a state system can be used by policy makers to evaluate proposed district consolidations in light of opportunity costs. Since the data inputs for the model are currently available in published records of state departments of education, it could easily be applied in other states.

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FOOTNOTES

1. These and other methodological differences have lead to a number of conflicting recommendations in the research dealing with the "optimal school district size" question. For example, Hickey (1969) has shown that over the past 40 years this literature contains an assortment of recommended figures with a range falling between 1,200 and 160,000 pupils per district (p. 10). His review also provides an excellent overview of the different criteria used in these studies.

TABLE 1. SUMMARY OF COST-SIZE FUNCTIONS FOR OREGON SCHOOL DISTRICTS

ANALYSIS OF VARIANCE

	Degrees of Freedom	Sum of Squares	Mean Square	F ratio
Regression	3	305693.54	101897.84	21210.31
Residual	138	662.97	4.80	
Total	141	306356.51		

REGRESSION EQUATIONS

$$Z = .04395 + .74763A_1 + .00029A_2 + .000000285A_3 \qquad (29)$$

$$(.01616)^* \quad (.00011)^* \quad (.00000025)^*$$

[Multiple R^2 = .99892 Standard Error of Regression = 2.19184]

$$C = 4395.0 + 747.63Q - .0029Q^2 + .00000000285Q^3 \qquad (26)$$

$$AC = 4395.0Q^{-1} + 747.63 - .0029Q + .00000000285Q^2 \qquad (27)$$

*Standard error of coefficient above.

TABLE 2. SELECTED VALUES FOR THE AVERAGE COST CURVE

ADM (Q)	Average Cost
100	$791.29
500	754.97
1,000	749.15
5,000	734.72
10,000	721.92
20,000	703.23
30,000	687.75
40,000	677.33
45,000	674.94
47,000	674.38
49,000	674.05
50,000	673.97
51,000	673.95
51,500	673.95
52,000	673.97
55,000	674.42
60,000	676.30
70,000	684.34
80,000	698.08
90,000	717.53
100,000	742.67

TABLE 3. COST ANALYSIS FOR PROPOSED THREE DISTRICT CONSOLIDATION

	ADM	Average Cost	Savings per ADM*	Total Annual Savings**
District One	21,521	698.61	17.04	366,717.00
District Two	9,682	722.68	41.11	398,027.00
District Three	3,356	739.53	57.96	194,513.00
Total	34,559	681.57		959,257.00

*Column three was calculated by subtracting 681.57 from the individual average costs.

**Column four was calculated by multiplying appropriate entries in column one by column three.

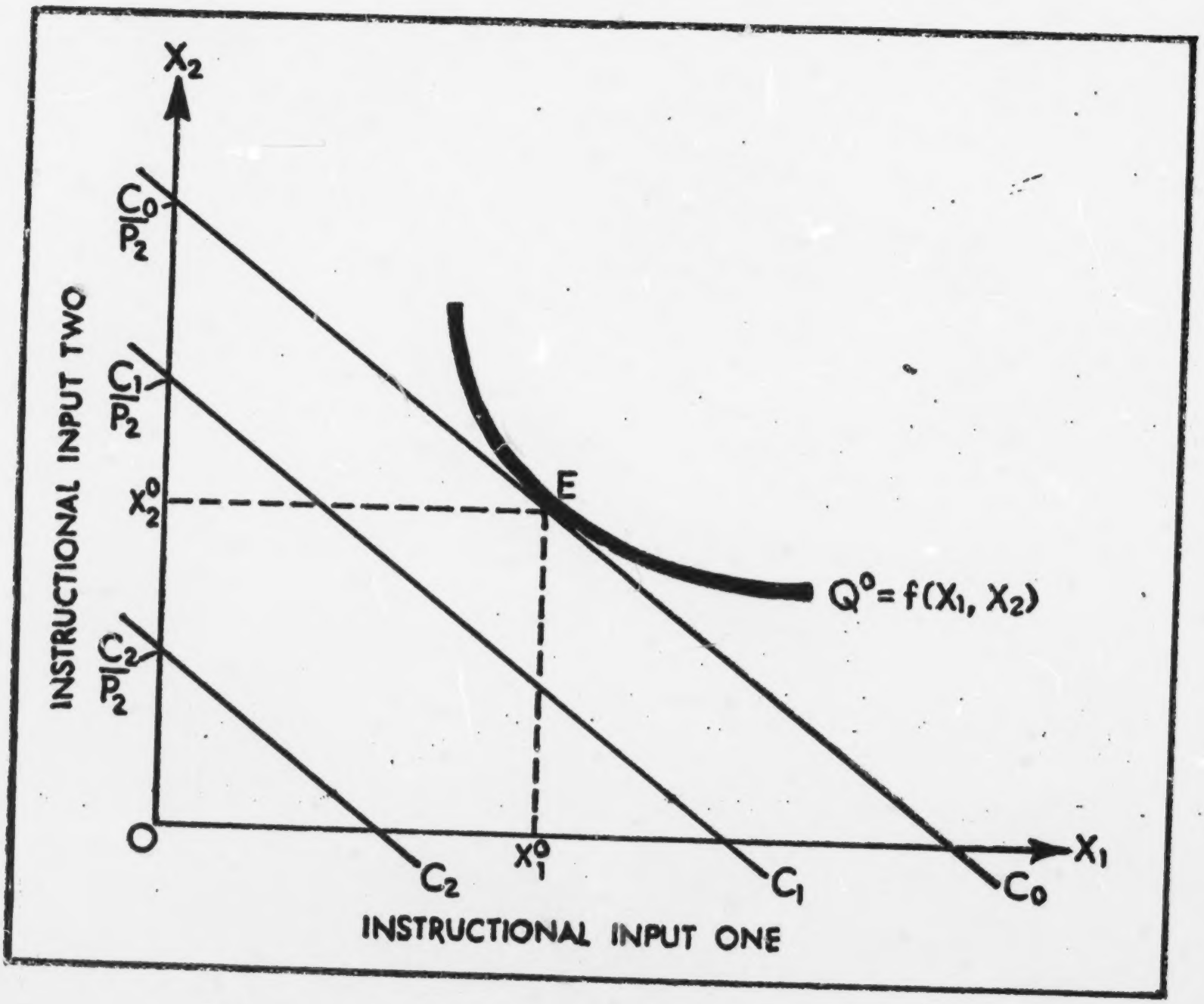

FIGURE 1. COST MINIMIZATION MODEL FOR AN INDIVIDUAL SCHOOL DISTRICT.

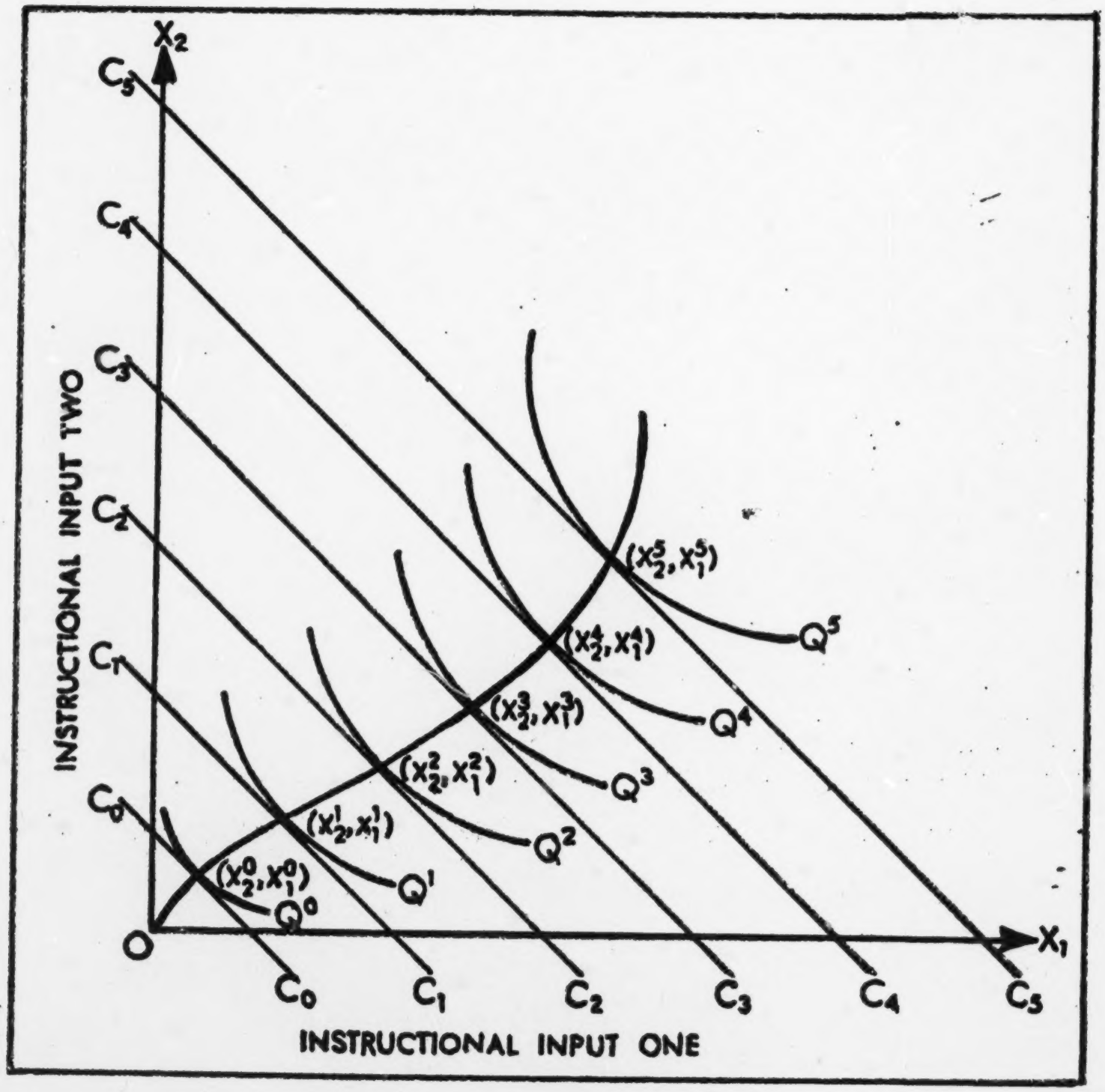

FIGURE 2. COST MINIMIZING EQUILIBRIA FOR ALL DISTRICTS

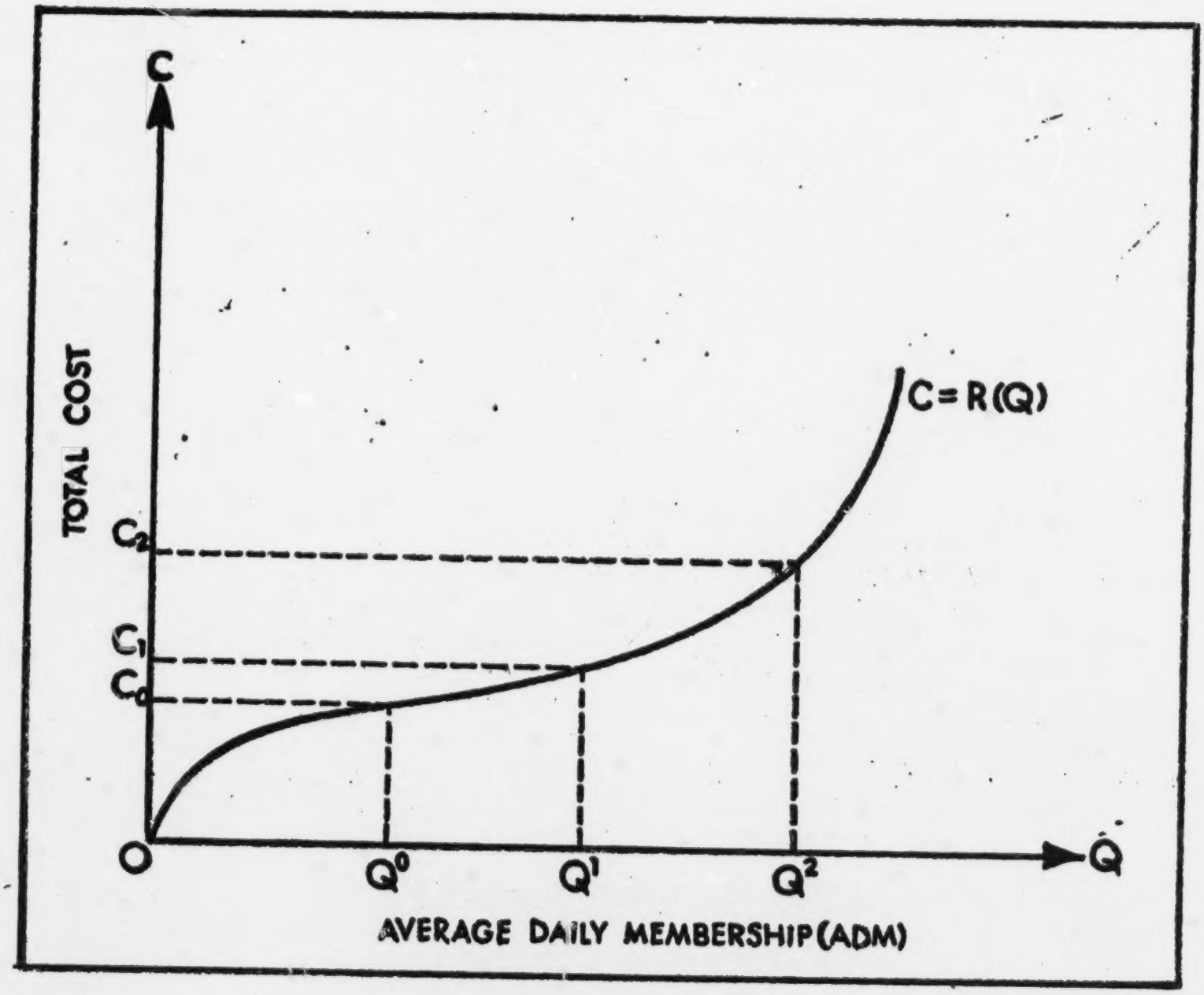

FIGURE 3. TOTAL COST AS A FUNCTION OF ADM

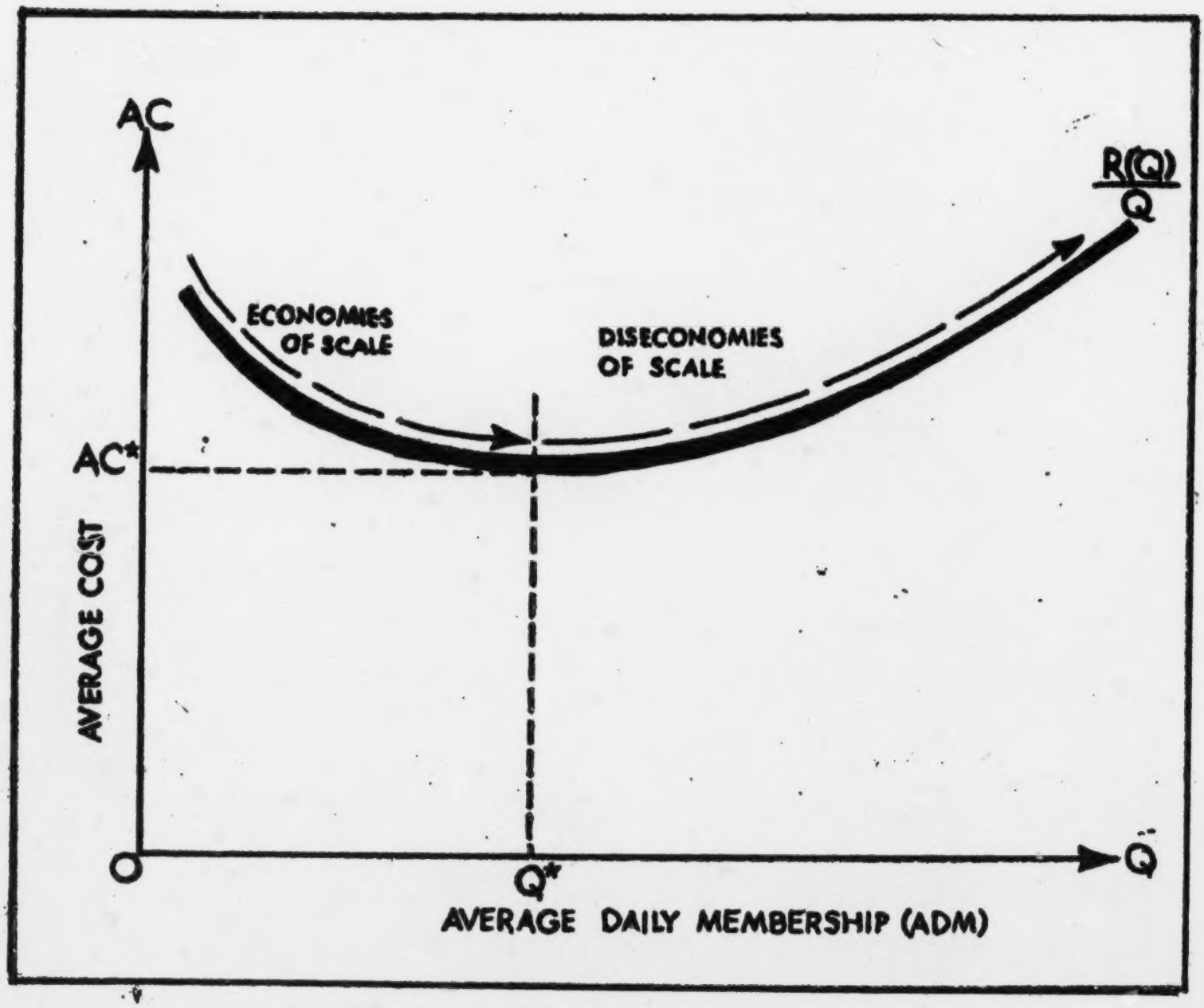

FIGURE 4. AN AVERAGE COST CURVE EXHIBITING AN OPTIMAL SCALE

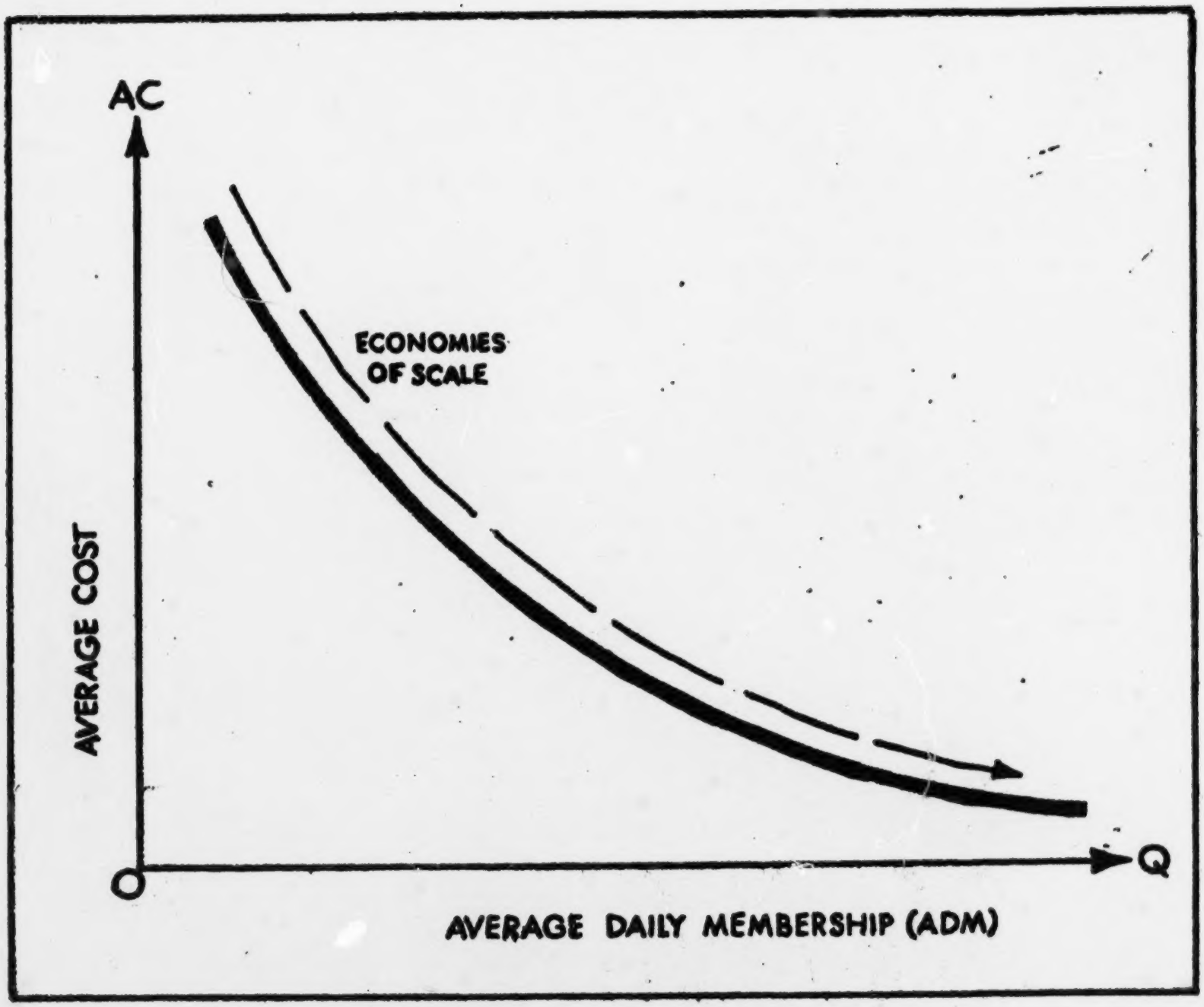

FIGURE 5. AN AVERAGE COST CURVE WHERE NO OPTIMAL SIZE EXISTS.

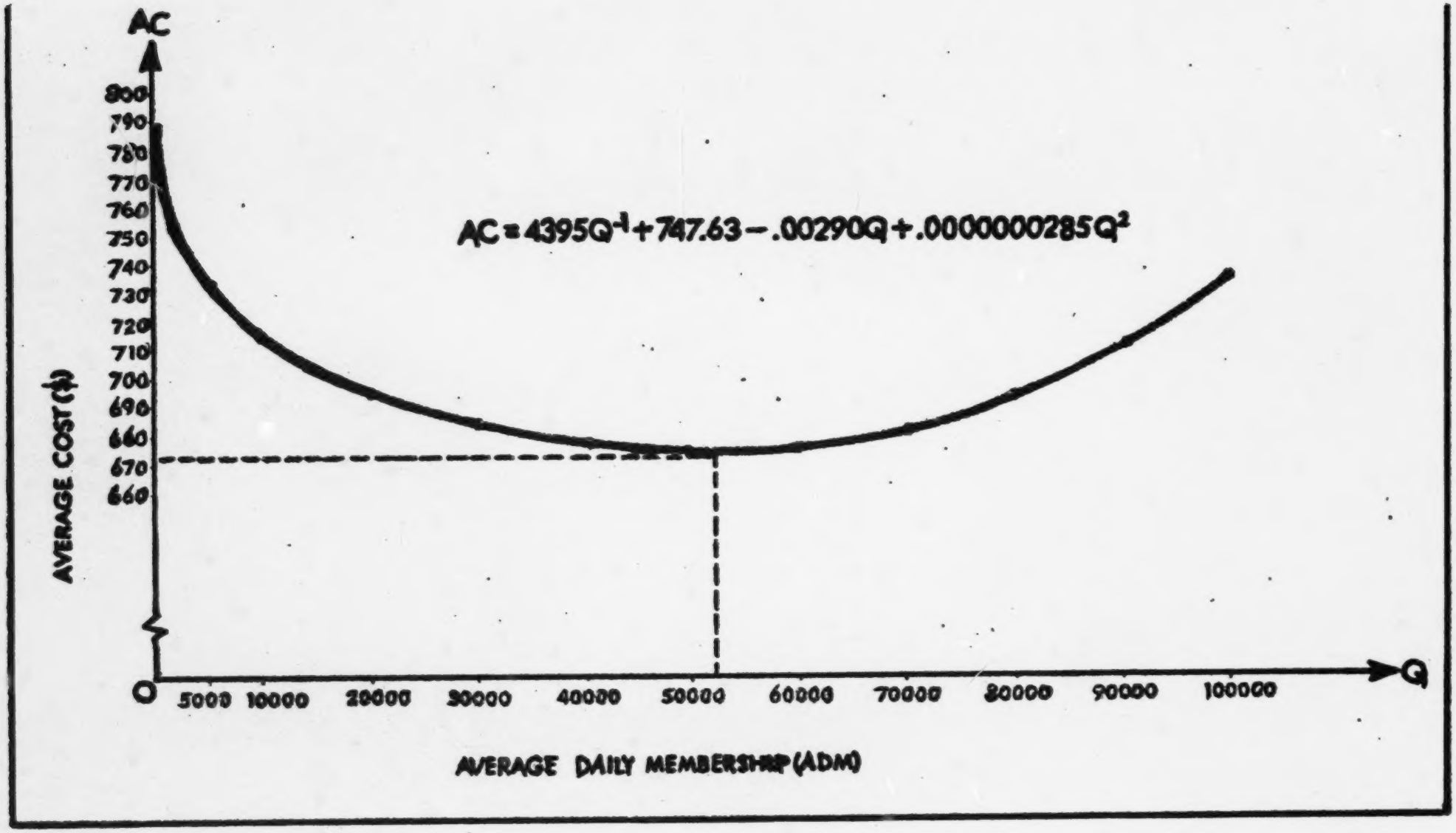

FIGURE 6. THE FITTED AVERAGE COST FOR OREGON SCHOOL DISTRICTS.